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# Compressed Channel Estimation for OFDM-Based Cognitive Radio

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**Abstract—***Wideband wireless channel is a time dispersive channel and becomes strongly frequency-selective, physical arguments and growing experimental evidence suggest that generally, the channel is composed of a few dominant taps and a large part of taps is approximately zero, that means wireless channels exhibit a sparse multipath structure for wideband wireless system. In this paper, we formalize the notion of multipath sparsity and present a novel approach based on Orthogonal Matching Pursuit (OMP) in the theory of Compressed Sensing (CS) to estimate sparse multipath channels for Cognitive Radio (CR). Proposed channel estimation scheme exploits a channel's delay-Doppler sparsity to reduce the number of pilots and, hence leads to increased spectral efficiency. The effectiveness of the proposed algorithm will be confirmed through comparisons with the existing Least Square (LS) channel estimation method.*

## 1. INTRODUCTION

The immense growth of wireless access technologies calls for more and more spectrum resources, but most of the spectrum bands are already allocated to specific licensed services. It has been observed that, in some locations or at some time period of the day, 60 percent of the allocated spectrum may be sitting idle [1]. Besides this a lot of licensed bands, such as those for TV broadcasting, are not utilized properly, resulting in spectrum wastage [2]. Federal Communications Commission (FCC) has recommended that significantly greater efficiency could be realized by developing wireless systems that can coexist with the Primary Users (PUs), generating minimal interference while taking advantage of the available spectrum resources [3]. A system that can reliably sense the spectral environment bandwidth, detects the presence and absence of active users and use the spectrum only if the communication does not interfere with active users is defined as the term cognitive radio system [4-8].

The underlying sensing and spectrum shaping capabilities of OFDM, together with its flexibility and adaptive nature, make it the best transmission technology for CR systems. By deactivating subcarriers utilized by the PUs, interference between the PUs and the unlicensed users, also called Secondary Users (SUs) can be mitigated [9,10]. However, the presence of deactivated subcarriers in the active subcarrier zone may possibly lead to non-contiguous sequence of the available subcarriers for the SUs and thereby complicate the design of efficient pilot symbols for channel estimation [9-12].

In [13], the pilot design is formulated as an optimization problem minimizing an upper bound related to the Mean Square Error (MSE), where the pilot indexes are obtained by solving a series of 1-dimentional low complexity problems. Manasseh *et al.* [14], proposed a pilot design scheme using convex optimization together with the cross-entropy optimization to minimize the MSE. Adaptation of parameters for wireless multicarrier-based CR systems is investigated in [15], where the cross-entropy method is demonstrated. However, all of them are based on the Least Square (LS) channel estimation.

Recently, applications of theory of CS to channel estimation, have shown that, by exploring the sparse nature of wideband multipath channels, improved channel estimation performance and reduced pilot overhead can be achieved. The sparse channel estimation for OFDM systems has been intensively studied [16], [17], and many CS algorithms such as OMP, compressive sampling matching pursuit, and basis pursuit, have been applied for OFDM channel estimation. In this paper we are extending compressed channel estimation to OFDM-based CR systems, which can improve the data rate and flexibility of SUs. Conventional methods for channel estimation are not able to exploit the inherent sparsity of the transmission channel which is due to the sparse distribution in space. In this paper, we will demonstrate that how CS theory provides a constructive way for exploiting the sparsity in order to reduce the number of pilots and get increased spectral efficiency.

The rest of this paper is organized as follows– Section II presents OFDM-based CR system model. This model plays a key role in subsequent developments in the paper and sets the stage for the application of CS theory to the channel estimation. In Section III, first brief review of the theory of CS is provided then we formalize the notion of sparse multipath channels and then apply the compressed channel estimation. In Section IV, we succinctly summarize the performance advantages of proposed algorithm over traditional LS-based channel estimation. Finally, we conclude the paper in Section

V by discussing some of the finer technical details pertaining to the results presented in the paper and future scope.

## 2. COGNITIVE RADIO SYSTEM MODEL

Non-Contiguous Orthogonal Frequency Division Multiplexing (NC-OFDM) system as shown in figure 1, keeps sensing the surrounding spectrum environment. The sub-carrier is set to activate when the spectrum is idle, this state is called “on”. When the spectrum is not available the subcarrier is set to zero, this state is called “off”. At the transmitter side, high-speed data stream first passes through the Quadrature Phase Shift Keying (QPSK) modulator and the modulated

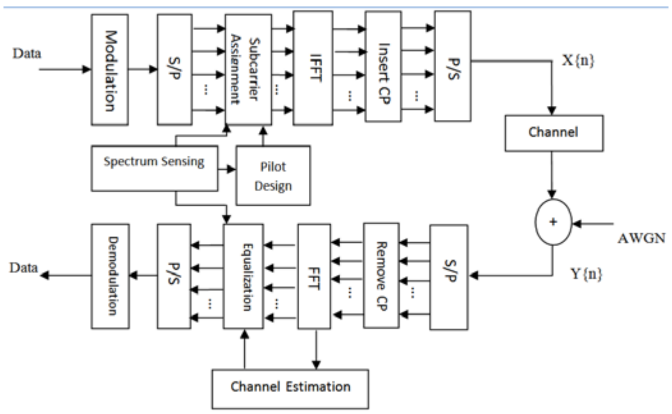


**Fig. 1: OFDM-based CR system model**

high-speed data stream is allocated into N lower rate subcarriers after the serial to parallel (S/P) conversion. Unlike the general OFDM systems where a serial data stream is assigned to all subcarriers, NC-OFDM system considered is in the control of sub-carrier state, which is on or off. The data stream is only assigned to the sub-carriers whose state is on, and for the sub-carriers whose state is off no data is transmitted. Pilot design is performed, based on the results of spectrum sensing. Note that here, ideal spectrum sensing is assumed, without any false alarm or missing detection. Pilots are inserted and N-point Inverse Fast Fourier transform (IFFT) is taken. In order to reduce the Inter Signal Interference (ISI) and Inter Carrier Interference (ICI), Cyclic Prefix (CP) is inserted into each of NC-OFDM symbol, and after the Parallel to Serial conversion (P/S), the data is recovered into a serial data stream and the signal is transmitted after the RF modulation,

At the receiver side, the received signal passes through the RF demodulator and after S/P converter the CP is removed. After the N-point Fast Fourier Transform (FFT), channel estimation is taken to combat multipath fading. The pilot symbol is removed and the data is read according to the sub-carrier state. At last, original data is recovered after QPSK demodulation.

In a broadband wireless communication system, the actual bandwidth of the system is usually larger than the coherence bandwidth and the channel is usually frequency selective fading channel.

The discrete-time channel model is:

$$h(n) = \sum_{l=0}^{l-1} h_l \delta(n-l) \tag{1}$$

Where the channel impulse response vector $h = [h_0, h_1, \ldots, h_{l-1}]^T$ remains unchanged in multiple OFDM symbol period of time reflects the slow time variation of the channel.

The vector is K-sparse if the number of its nonzero elements is K, Discrete Fourier Transform (DFT) size is N and pilot sub-carriers are P. The CP length is greater than the maximum possible path delay. OFDM symbol data X(n) contains mapping signals and pilot signals. At the receiver, we assume perfect timing and frequency synchronization. After removing CP, we apply Discrete Fourier Transform (DFT) to the received time-domain signal $y_n$ for $n \in [0, N-1]$ to obtain for $k \in [0, N-1]$. The received signal is N×1 sample vector:

$$y = XH + Z = XWh + Z \tag{2}$$

where X=diag(X(1), X(2),…, X(N)), N×1 matrix H is the sample value of channel frequency response. N×1 matrix Z is Additive White Gaussian Noise (AWGN). N×L matrix W is the first L columns of the DFT transform matrix:

$$W = \begin{bmatrix} w^{00} & \cdots & w^{(l-1)0} \\ \vdots & \ddots & \vdots \\ w^{0(N-1)} & \cdots & w^{(l-1)(n-1)} \end{bmatrix} \tag{3}$$

Where $w^{nl} = e^{-j\frac{2\pi nl}{N}}$ .

P×N matrix S selects the location of P pilot from the N subcarriers. N×N matrix S selects P rows corresponds to the pilot position from the unit matrix. The pilot signal recieved is:

$$y_p = X_p H_p + Z_p = X_p W_p h + Z_p \tag{4}$$

where P × 1 matrix $y_p$=Sy , P × P matrix $X_p$=SXS’ , P × L matrix $W_p$= SW , P × 1 matrix $n_P$=S.In(4)$y_p$, $X_p$ and $W_p$ are known. We can reobtain system channel relation obtained in (4) by delay Doppler domain expression of the channel coefficients H.

$$H_{lk} = \sum_{m=0}^{K-1} \sum_{i=0}^{L=1} F[m,i] e^{-j2\Pi(\frac{km}{K}-\frac{l_i}{i})} \tag{5}$$

Where

$$F[m,i] = X_h(m,i) A^{*}_{\Upsilon,g}(m, i/N_r) \tag{6}$$

With the cross ambiguity function [21] $A_{\gamma,g}(m,\varepsilon)=\sum_{n=\infty}^{-\infty} \gamma[n] g[n-m]\, e^{-2\Pi\varepsilon n}$. The purpose of channel estimation is to estimate the channel frequency response H.

## 3. COMPRESSED CHANNEL ESTIMATION

CS theory suggests that if the signal is compressible or in a transform basis is sparse, the high dimensional signal can be projected onto a lower dimensional space with an observation matrix which is uncorrelated with the transform basis, and then the original signal will be reconstructed with high probability, by solving an optimization problem with the small amount of projections. In the CS model, the signal x is not directly measured, but projected onto the observation matrix $\Phi$=[$\Phi$1,$\Phi$2,…$\Phi$M], and then the sampling vector y is obtained. The matrix form is

$$y = \Phi x \quad (7)$$

where x is an N × 1 vector, $\Phi$ is an M × N matrix, and y is an M × 1 sampling vector.

If signal x is sparse in the transform basis $\psi^T = [\psi_1, \psi_2, .. \psi_N]$ , it can be expressed as follows:

$$x = \sum_{k=1}^{N} \psi_k \alpha_k = \psi\alpha \quad (8)$$

where α is an N ×1 vector, ψ is an N × N sparse vector. (8) is substituted into (7):

$$y = \Phi x = \Phi\psi\alpha = \Theta\alpha \quad (9)$$

where M × N matrix $\Phi=\psi\Theta$. The observation dimension M is far less than the signal dimension N, so the signal x can't be solved with M sampling values of y. As α in (9) is sparse, with the sparse decomposition algorithm based on the theory of sparse decomposition, α can be solved by the inverse problem of solving (9), and be substituted into (8) to obtain the signal x. Consider the baseband channel model of (4). If we want to guarantee accurate channel estimation, Θ must satisfy two conditions.

*1) Restricted Isometry Property (RIP)* [18]
For signal reconstruction to be successful, Θ must satisfy

$$(1-\delta_s)\, ||h||_2^2 \le ||\Theta h||_2^2 \le (1+\delta_s)\, ||h||_2^2 \quad (10)$$

From CS perspective, research on the RIP of the training sequence has two important purposes. First, RIP-based training sequence is a sufficient condition to robust probe sparse channel dominant taps. Furthermore, in the process of error performance analysis, RIC of training sequences play important role to improve lower bound.

*2) Lower bound of length of training sequence Θ* [19]
Due to the channel fading and noise, to determine the length of training sequence Θ is important in terms of both spectrum efficiency and estimation robustness. Therefore, the length N of Θ must satisfy

$$N \ge C \cdot K \cdot (\log L)^4\, \mu_x^2 \quad (11)$$

where C is a constant and $\mu_x = \sqrt{L}\, \max_{i,j}|X_{i,j}|$ which is known as the maximum coherence between the $i_{th}$ column and $j_{th}$ column of Θ.

OMP algorithms suggests the reconstruction under the conditions of a given iteration number, as the iterative process is forced to stop, OMP algorithm needs a lot of linear measurement to ensure accurate reconstruction. The basic idea of the OMP algorithm is to select the columns of Θ with greedy iterative algorithm, make sure the correlative value between the columns selected in each iteration and the current redundant vector is maximum, and then subtract the correlative value from the sampling vector and repeat iteration until the number of iterations achieves the sparse degree *K*.

Algorithm 1. OMP-CS

**Input:** sensing matrix Θ, sampling vector y , sparse degree K ;
**Output:** the K-sparse approximation $\hat{\alpha}$ of α ;
**Initialization**: the residual $r_0$=y , index set $\Lambda_0=\Phi$ t=1;
Step 1: find the maximum value of the inner product of residual r and the column of sensing matrix $\theta_j$ the corresponding foot mark is λ, $\lambda_t = \arg_{j=1..N} \max |\langle r_{t-1}\, \theta_j \rangle|$
Step 2: renew the index set $\Lambda_t = \Lambda_{t-1} \cup \{\lambda_t\}$ , the sensing matrix $\Theta_t = [\Theta_{t-1}, \theta_{\lambda_t}]$
Step 3: solve $\hat{\alpha}_t = \arg\min \| y - \Theta_t, \hat{\alpha} \|_2$ by least-square method;
Step 4: renew the residual $r_t = y - \Theta_t \hat{\alpha}_t$ t= t+1
Step 5: **if** t > K , stop the iteration,
**else** do step 1.

OMP algorithm selects an atom in each iteration to update the atom collection, which will certainly pay a large time for reconstruction. The number of iterations is closely related to sparse degree K and the number of samples M, with their increase, time consumption will also increase significantly.

Problem with algorithm 1 is that it is not adaptive, pre-estimate of the sparse degree of the sparse signal is needed and the reconstruction accuracy is not satisfactory. In reality, the sparse degree of the sparse channel is usually unknown. Nam Naguyen *et al.* [20] proposed an algorithm in order to improve the accuracy of reconstruction, and make the algorithm adaptive. The signal reconstructed does not require any a priori information on signal sparse degree. The process is as follows:

Algorithm 2. Extended OMP-CS

**Input:** Sampling matrix Θ , Sampled vector y , step size s ;
**Output:** A K-sparse approximation $\hat{\alpha}$ of the input signal ;
**Initialization** $\hat{\alpha} = 0$; $r_0 = y$; $F_0 = \Phi$; I = s; k = 1; stage = 1
*repeat*
$S_k = \max(|\Theta^* r_{k-1}, I|)$ (Preliminary Select)
$C_k = F_{k-1} \cup S_k$ ( Make Candidate List)

F = max( $|\Theta\ r_{k-1}|$, I ) (Final Select)
F= y- $\Theta\Theta_F^*$ (Compute Residue)
*if* halting condition is true - quit the iteration;
*else*
*if* $\|r\|_2 \geq \|r_{k-1}\|_2$ - stage switching
stage = stage+ 1, j = j +1 (Update the stage index)
I = stage × s (Update the size of finalist)
*else*
$F_k$=F (Update the finalist)
$r_k$=r (Update the residue)
k=k+1
*end*
*end*
*repeat* until halting condition is true;
Output $\alpha_F = \Theta_F^*$ y

In the ExtOMP-CS algorithm, one key issue is how to choose the step size s. To avoid overestimation, the best choice is certainly s =1 if sparse degree K is unknown. However, a trade-off is there between step size s and the recovery speed, as smaller s requires more iterations. Unlike the OMP-CS algorithm, the iteration times of ExtOMP-CS algorithm is not certain and is related to step size s. The proposed algorithm introduces the idea of stage, reconstruction of signal is divided into several stages, and each stage contains a number of iterations. Therefore, computational complexity and computational time are higher in the ExtOMP-CS algorithm than OMP-CS algorithm.

If N-length training sequence X satisfies the RIP and $\delta_{2S} \leq \sqrt{2}-1$, for any 2S-sparse channel vector h, ExtOMP-CS algorithm produces the channel estimator $\hat{h}$ that satisfies

$$\|h-\hat{h}\|_2 \leq C \max\{\varepsilon, 1/\sqrt{S}\ \|h-\hat{h}_{2S}\|_1 + \|z\|_2\} \quad (12)$$

for a given parameter ε . And $h_{2S}$ is a best 2S-sparse approximation to h.

For practical channels as well as transmit and receive pulses, the function F[m, i] in (6) is effectively supported in a sub domain of the delay-Doppler plane. This allows us to perform a sub sampling in the time-frequency domain. From (4), the receiver calculates channel coefficient estimates $\hat{H}_{jk}$ at the pilot positions (l, k) according to

$$H(l,k)=\hat{H}(l,k) + \frac{n(l,k)}{p(l,k)} \quad (13)$$

Thus, H(l,k) is known up to the additive noise terms $n_{l,k}/p_{l,k}$. We can hence use the extended OMP-CS algorithm to obtain an estimate of F[m, i]. From this estimate, estimates of all channel coefficients $H_{l,k}$ are finally obtained. Thus, CS-based channel estimation is computationally feasible, albeit more complex than classical LS channel estimation.

Regarding the choice of the pilot positions (l, k) ϵ $\mathcal{P}$, we recall that these positions correspond to $|\mathcal{P}|$ indices within the index range of the channel vector h. To be consistent with the CS framework, we select these positions uniformly at random. For good approximation quality, the number of pilots should satisfy condition (11) where μ=1. This bound is not useful for actually determining $|\mathcal{P}|$ because of the constant C. However, the bound suggests that the required number of pilots scales only linearly with P of channel paths and the sparsity parameter K. In practice, the pilot positions will be randomly chosen and communicated to the receiver, only once before the beginning of data transmission. With high probability, they will lead to good performance for arbitrary channels with at most P paths.

## 4. SIMULATION RESULTS AND DISCUSSION

In this section we present numerical results to compare the performance of the proposed OMP-CS channel estimation method with that of classical LS channel estimation. We are considering an OFDM-based CR system with P = 1024 subcarriers, after spectrum sensing without any false alarm or missing detection and deactivating those subcarriers occupied by PUs, we assume that there are 512 remaining OFDM subcarriers for SUs, including three noncontiguous subcarrier blocks, i.e., {1, 2, . . . , 256}, {513, 514, . . . , 640} and {897, 898, . . . , 1024}, with the number of subcarriers in each block being 256, 128, and 128, respectively. We now evaluate the channel estimation performance using the designed pilot patterns. The Mean Square Error (MSE) performance for channel estimation and the Bit Error Rate (BER) performance for data detection are shown in Figures 2 and 3, respectively.

*A. Estimation Error*

The MSE performance of the proposed estimation method will be evaluated by simulations and compared with the MSE performance of LS. It is obvious that smaller MSE means more accurate channel estimation and vice versa. MSE is defined as

$$\text{MSE} = 1/M \sum_{m=1}^{M} \left\| h - \hat{h}_m \right\|_2^2 \quad (14)$$

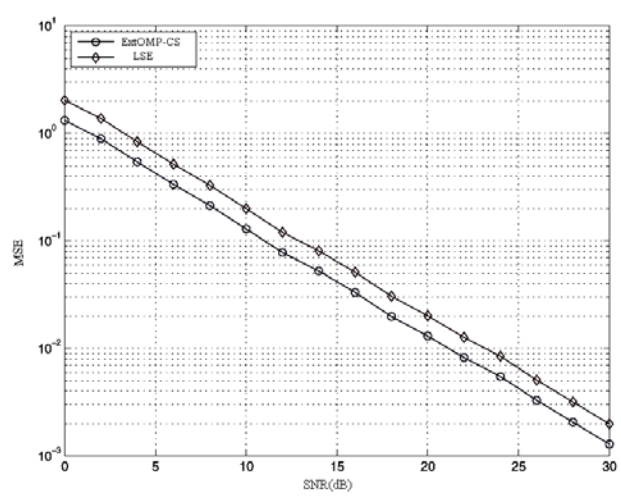

**Fig. 2: Performance of the channel estimate MSE for different SNRs**

Fig. 2 depicts the MSE versus SNR. It is seen that the CS-based proposed method significantly outperforms the least-squares method. The extremely poor performance of the least-squares method is due to the fact that the Shannon sampling theorem is violated by the pilot grid. In contrast, the CS based method is able to produce reliable channel estimates even far below the Shannon sampling rate. Compared with the LS method with 1024 pilots, we observe only a relatively small performance degradation of the CS-based method with 511 pilots.

*B. Bit Error Rate (BER)*

Next, we make comparison of the BER performance of the proposed pilot symbols. Figure 3 shows BER performance of two schemes. The results show improved BER performance of the proposed design over the conventional LS design.

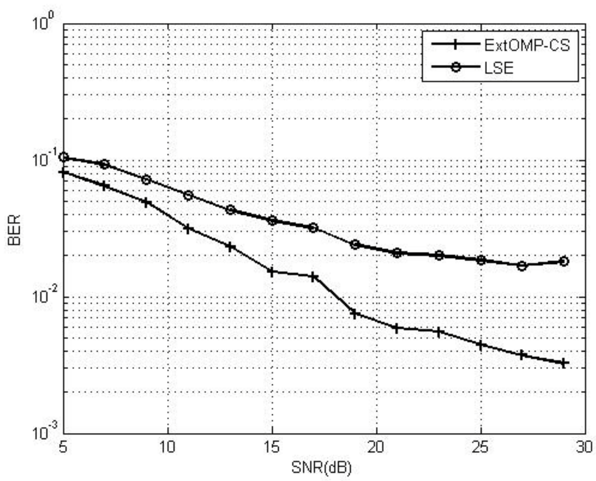


**Fig. 3: Comparison of the BER performances for different SNRs**

## 5. CONCLUSION

We proposed a channel estimation technique based on the theory of CS. Our results demonstrate that CS makes it possible to exploit the delay-Doppler sparsity of wireless channels for a reduction of the number of pilots required for channel estimation within multicarrier systems. In this paper, we have introduced a novel sparse channel estimation method OMP-CS based on the CS theory. The extended OMP-CS method has both advantages of the greedy algorithm and convex program algorithm. On future work, we will consider an adaptive OMPCS channel estimation method which senses the random noise and other unexpected interferences. The complexity of our proposed design is slightly higher than that of the conventional scheme but combatively lower than that of the exhausted search scheme.

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